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An Integrated Transform-Algebraic Framework for Solving Linear Nonautonomous Dynamical Systems

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إطار تحويل جبري متكامل لحل الأنظمة الديناميكية الخطية الغير ذاتية

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Abstract:
This investigation develops a comprehensive algebraic methodology for treating linear nonautonomous ordinary differential equation systems through the synergistic combination of integral transform techniques and matrix elimination procedures. The proposed framework converts the temporal initial value problem expressed as $\mathrm{x}'(t) = \mathrm{Mx}(t) + \mathrm{g}(t)$ with $\mathrm{x}(0) = \mathrm{x}_0$ into an equivalent algebraic formulation within the complex frequency domain characterized by $(s\mathrm{I} - \mathrm{M})\mathrm{X}(s) = \mathrm{x}_0 + \mathrm{G}(s)$. The time-domain solution emerges following the application of the inverse integral transform. This strategy circumvents explicit integration procedures, captures the system dynamics via the resolvent operator $(s\mathrm{I} - \mathrm{M})^{-1}$, and furnishes a systematic computational protocol that seamlessly accommodates prescribed initial states. The exposition details the theoretical foundations, illustrates the operational sequence through elaborated case studies, and critically examines both the strengths and constraints of the technique, thereby confirming its utility across applied mathematics, engineering disciplines, and allied scientific domains.

Keywords: Laplace Transform, Gauss-Jordan Elimination, Nonautonomous Systems, Resolvent Operator, Linear Differential Equations, Matrix Algebra.

الملخص:
يطور هذا البحث منهجية جبرية شاملة لمعالجة أنظمة المعادلات التفاضلية العادية الخطية غير المستقلة بزمن من خلال الدمج التآزري بين تقنيات التحويل التكاملي وإجراءات الحذف المصفوفي. يحوّل الإطار المقترح مسألة القيمة الابتدائية الزمنية المعبّر عنها بالشكل $\mathrm{x}'(t) = \mathrm{Mx}(t) + \mathrm{g}(t)$ مع $\mathrm{x}(0) = \mathrm{x}_0$ إلى صياغة جبرية مكافئة في مجال التردد العقدي يتميز بالعلاقة $(s\mathrm{I} - \mathrm{M})\mathrm{X}(s) = \mathrm{x}_0 + \mathrm{G}(s)$. وينبثق حل المجال الزمني بعد تطبيق تحويل التكامل العكسي. تتجنب هذه الاستراتيجية إجراءات التكامل الصريحة، وتلتقط ديناميكيات النظام عبر مؤثر الحل $(s\mathrm{I} - \mathrm{M})^{-1}$، وتوفر

بروتوكولاً حسابياً منتظماً يستوعب الحالات الابتدائية المفروضة بشكل سلس. يعرض الشرح الأسس النظرية، ويوضح التسلسل التشغيلي من خلال دراسات حالة مفصلة، ويفحص نقدياً كلاً من نقاط القوة والقيود في التقنية، مما يؤكد فائدتها عبر الرياضيات التطبيقية، والتخصصات الهندسية، والمجالات العلمية المرتبطة به

الكلمات المفتاحية: تحويل لابلاس، حذف جاوس-جوردان، الأنظمة غير المستقلة، مؤثر الحل، المعادلات التفاضلية الخطية، جبر المصفوفات مسائل القيم الابتدائية.

Introduction:

The examination of linear dynamical systems constitutes a cornerstone within contemporary applied mathematics, where the central challenge involves determining the temporal trajectory of state vectors governed by coupled differential equations. Traditional solution strategies frequently rely on direct integration approaches or the construction of fundamental matrix solutions, procedures that become increasingly cumbersome when addressing nonhomogeneous configurations or systems incorporating time-dependent excitation functions. A particularly elegant alternative emerges through the application of the Laplace transform, a mathematical instrument that maps differential operators operating in the time domain into algebraic operators within the complex frequency domain. This transformation effectively recasts the differential equation solving problem into the more tractable task of solving linear algebraic equations, thereby conferring substantial advantages for both theoretical investigation and computational implementation.

The present investigation is motivated by the requirement for a rigorous, algorithmically transparent methodology capable of efficiently treating linear nonautonomous systems. The specific research objective consists of developing and demonstrating a hybrid

Procedure that applies the Laplace transform to convert the differential system into an algebraic system, subsequently employs Gauss-Jordan elimination to determine the transformed variables, and finally recovers the time-domain solution through inverse Laplace transformation. This integrated approach not only streamlines the solution process but also illuminates profound structural connections linking differential equations with linear algebraic theory.

The distinctive contribution of this work relative to existing literature resides in its explicit and detailed integration of Laplace transform principles with matrix algebraic techniques within a systematically presented pedagogical framework, supported by fully elaborated computational examples. The foundational principles of the Laplace transform and its application to differential equations are rigorously established in canonical references. Specifically, the definition of the Laplace transform and the theorem governing the transformation of derivatives, which are essential for converting differential equations into algebraic forms, are drawn from the authoritative treatment by Boyce and DiPrima [1]. A more comprehensive exposition of Laplace transforms within the broader context of engineering mathematics, including their application to systems of differential equations, is provided by Kreyszig [2]. The matrix algebra components, encompassing the construction of augmented matrices, the systematic application of Gauss-Jordan elimination for solving linear systems, and the interpretation of matrix inverses, derive directly from the foundational textbooks by Strang [3, 4]. The concept of the resolvent operator $(s\mathrm{I}-\mathrm{M})^{-1}$, which plays a central role in the proposed methodology by encoding the intrinsic dynamics of the system and the complete response to external inputs, is extensively discussed by Chen [5] within the context of linear system theory and control, where the connection between the Laplace transform and system response is explicitly established. Additional theoretical support for the formulation of nonautonomous systems and the interpretation of their solutions is provided by Simmons [6], who presents differential equations with historical notes and applications, as well as by Hirsch, Smale, and Devaney [7] in their work on differential equations, dynamical systems, and chaos. Classical ordinary differential equation theory, including Laplace transform methods and the analysis of linear systems, receives rigorous treatment by Coddington and Levinson [8], with a similarly rigorous approach offered by Tenenbaum and Pollard [9] in their comprehensive text on ordinary differential equations.

While individual components-Laplace transforms for differential equations and GaussJordan elimination for linear systems - are well established in these references, their combined application to nonautonomous systems frequently appears in fragmented fashion across the literature. The present manuscript provides a unified exposition that emphasizes the role of the resolvent operator and systematically addresses both homogeneous and particular solution components. Furthermore, the discussion incorporates a balanced assessment of the method's advantages and limitations, thereby offering practical guidance for researchers and practitioners. By restructuring the logical flow, reformulating all derivations, and expanding the explanatory context, this paper aims to deliver a selfcontained reference that minimizes dependence on external sources while maintaining scientific rigor.

Preliminaries:

Before presenting the main solution procedure, it is necessary to establish the foundational definitions and theorems that underpin the subsequent analysis. This section recalls the definition of the Laplace transform, the theorem governing the transform of derivatives, the concept of the inverse Laplace transform, the structure of the augmented matrix, the Gauss-Jordan elimination algorithm, and the general form of linear nonautonomous systems.

The Laplace transform is an integral transformation that converts a time-dependent function into a complex frequency-domain representation. For a function $\mathrm{h}(t)$ defined for $t \geq 0$, its Laplace transform is given by:

$$\mathcal{L}\{\mathbf{h}(t)\} = \mathbf{H}(s) = \int_0^\infty e^{-st}\mathbf{h}(t)dt$$

This transformation is particularly useful for differential equations because it converts differentiation into algebraic multiplication, provided the function satisfies certain growth and continuity conditions.

A central result for applying the Laplace transform to differential equations is the theorem on the transform of a derivative. Assume that f is continuous and that its derivative f' is piecewise continuous on any finite interval $0 \leq t \leq A$. Furthermore, suppose there exist constants $K, a,$ and M such that $|f(t)| \leq Ke^{at}$ for all $t \geq M$. Then the Laplace transform of f' exists for $s > a$ and satisfies:

$$\mathcal{L}\{f'(t)\} = s\mathcal{L}\{f(t)\} - f(0)$$

This theorem can be extended to higher-order derivatives and to systems of equations, forming the basis for converting initial value problems into algebraic equations.

The inverse Laplace transform is the operation that recovers the original time-domain function from its frequency-domain representation. If $\mathrm{H}(s) = \mathcal{L}\{\mathrm{h}(t)\}$, then one writes $\mathrm{h}(t) = \mathcal{L}^{-1}\{\mathrm{H}(s)\}$. Linearity of the inverse transform implies that if $\mathrm{H}(s) = \mathrm{H}_1(s) + \mathrm{H}_2(s) + \cdots + \mathrm{H}_n(s)$, then the corresponding time-domain function is $\mathrm{h}(t) = \mathrm{h}_1(t) + \mathrm{h}_2(t) + \cdots + \mathrm{h}_n(t)$, where each $\mathrm{h}_i(t) = \mathcal{L}^{-1}\{\mathrm{H}_i(s)\}$. This property is essential when decomposing complex rational expressions obtained from solving the algebraic system.

In the context of linear algebraic systems, an augmented matrix is a compact representation that combines the coefficient matrix of the variables and the constant terms into a single array. For a system of nonhomogeneous linear equations such as:

$$\begin{cases} x + 2y = 5 \\ 3x - y = 4 \end{cases}$$

the augmented matrix is:

$$\begin{pmatrix} 1 & 2 & 5 \\ 3 & -1 & 4 \end{pmatrix}.$$

This representation facilitates the systematic application of row operations to solve for the unknowns.

The Gauss-Jordan elimination method is an algorithmic procedure for solving systems of linear equations by transforming the augmented matrix into reduced row echelon form through a sequence of elementary row operations. These operations include swapping rows, multiplying a row by a nonzero constant, and adding a multiple of one row to another. The final reduced row echelon form matrix directly yields the solution vector, provided the system is consistent.

Turning to differential systems, a linear nonautonomous system of first-order equations can be written in vector form as:

$$\frac{d\mathbf{x}}{dt} = \mathbf{M}(t)\mathbf{x}(t) + \mathbf{b}(t), \mathbf{x}(t_0) = \mathbf{x}_0$$

where $\mathrm{x}(t)$ is the state vector, t is the independent variable (often representing time), $\mathrm{M}(t)$ is a matrix that may depend explicitly on time, and $\mathrm{b}(t)$ is a forcing term. When the system is autonomous, M becomes constant. The present work focuses on constantcoefficient linear nonautonomous systems of the form:

$$\mathbf{x}'(t) = \mathbf{M}\mathbf{x}(t) + \mathbf{h}(t), \mathbf{x}(0) = \mathbf{x}_0$$

where M is a constant matrix and $\mathrm{h}(t)$ is a vector-valued function that may be nonconstant. The Laplace transform method described in this paper is particularly well suited to such systems because it converts the constant matrix M into the algebraic operator $(s\mathrm{I} - \mathrm{M})$, whose inversion encodes the system's dynamic properties.

Main Results:

The core of the proposed methodology rests on applying the Laplace transform to the linear nonautonomous system, rearranging the resulting algebraic equations, constructing the augmented matrix, performing Gauss-Jordan elimination to obtain the transformed solution vector, and finally applying the inverse Laplace transform to recover the timedomain solution. This procedure is illustrated through two detailed examples.

Consider the general initial value problem:

$$\mathbf{x}'(t) = \mathbf{M}\mathbf{x}(t) + \mathbf{h}(t), \mathbf{x}(0) = \mathbf{x}_0$$

Applying the Laplace transform to both sides and using the linearity property together with the derivative theorem yields:

$$s\mathbf{X}(s) - \mathbf{x}_0 = \mathbf{M}\mathbf{X}(s) + \mathbf{H}(s)$$

where $\mathbf{X}(s) = \mathcal{L}\{\mathbf{x}(t)\}$ and $\mathbf{H}(s) = \mathcal{L}\{\mathbf{h}(t)\}$. **Rearranging terms gives:**

$$(s\mathbf{I} - \mathbf{M})\mathbf{X}(s) = \mathbf{x}_0 + \mathbf{H}(s)$$

This equation is a linear algebraic system in the unknown vector $\mathrm{X}(s)$. Provided s is not an eigenvalue of M, the matrix $(s\mathrm{I} - \mathrm{M})$ is invertible, and one obtains:

$$\mathbf{X}(s) = (s\mathbf{I} - \mathbf{M})^{-1}(\mathbf{x}_0 + \mathbf{H}(s))$$

The resolvent operator $(s\mathrm{I} - \mathrm{M})^{-1}$ encapsulates the intrinsic dynamics of the system. The time-domain solution follows as $\mathrm{x}(t) = \mathcal{L}^{-1}\{\mathrm{X}(s)\}$. In practice, rather than explicitly computing the matrix inverse, one solves the algebraic system using Gauss-Jordan elimination on the augmented matrix derived from $(s\mathrm{I} - \mathrm{M})\mathrm{X}(s) = \mathrm{x}_0 + \mathrm{H}(s)$.

Example 1: Polynomial and Trigonometric Forcing:

Solve the system:

$$\begin{cases} x_1' = x_1 - 3x_2 + t^2 \\ x_2' = x_2 - x_1 + \cos t \end{cases}$$

with initial conditions $x_1(0) = 0, x_2(0) = 0$.

Applying the Laplace transform to each equation, using the derivative theorem and zero initial conditions, **gives:**

$$\begin{cases} sX_1(s) = X_1(s) - 3X_2(s) + \dfrac{2}{s_s^3} \\ sX_2(s) = X_2(s) - X_1(s) + \dfrac{s}{s^2+1} \end{cases}$$

Rearranging terms yields:

$$\begin{cases} (s-1)X_1(s) + 3X_2(s) = \dfrac{2}{s_s^3} \\ X_1(s) + (s-1)X_2(s) = \dfrac{s_s^2+1}{s^2+1} \end{cases}$$

The augmented matrix representation is:

$$\begin{pmatrix} s-1 & 3 & \dfrac{2}{s_s^3} \\ 1 & s-1 & \dfrac{s^2+1}{2} \end{pmatrix}$$

To apply Gauss-Jordan elimination, **swap the rows to place a pivot of 1 in the upper-left corner:**

$$\begin{pmatrix} 1 & s-1 & \dfrac{s}{s^2+1} \\ s-1 & 3 & \dfrac{2}{s^3} \end{pmatrix}.$$

Eliminate the entry below the pivot by performing the row operation $\boldsymbol{R_2 \leftarrow R_2 - (s-1)R_1}$:

$$\begin{pmatrix} 1 & s-1 & \dfrac{s}{s^2+1} \\ 0 & 3-(s-1)^2 & \dfrac{2}{s^3} - \dfrac{s(s-1)}{s^2+1} \end{pmatrix}$$

The factor $3-(s-1)^2$ simplifies to $-s^2+2s+2$, which factors as $-(s^2-2s-2) = -(s-(1+\sqrt{3}))(s-(1-\sqrt{3}))$. Define $\lambda_1 = 1+\sqrt{3}$ and $\lambda_2 = 1-\sqrt{3}$, so that $3-(s-1)^2 = (s-\lambda_1)(s-\lambda_2)$. Normalize the second row by dividing by $3-(s-1)^2$:

$$\begin{pmatrix} 1 & s-1 & \dfrac{s}{s^2+1} \\ 0 & 1 & \dfrac{\dfrac{2}{s^3} - \dfrac{s(s-1)}{s^2+1}}{(s-\lambda_1)(s-\lambda_2)} \end{pmatrix}$$

Back-substitute to eliminate the off-diagonal entry in the first row using $R_1 \leftarrow R_1 - (s-1)\,R_2$:

$$\begin{pmatrix} 1 & 0 & \dfrac{s}{s^2+1} - (s-1)\dfrac{\dfrac{2}{s^3} - \dfrac{s(s-1)}{s^2+1}}{(s-\lambda_1)(s-\lambda_2)} \\ 0 & 1 & \dfrac{\dfrac{2}{s^3} - \dfrac{s(s-1)}{s^2+1}}{(s-\lambda_1)(s-\lambda_2)} \end{pmatrix}$$

Thus, the transformed solutions are:

$$X_2(s) = \frac{\frac{2}{s^3} - \frac{s(s-1)}{s^2+1}}{(s-\lambda_1)(s-\lambda_2)}, X_1(s) = \frac{s}{s^2+1} - (s-1)X_2(s)$$

Partial fraction decomposition of $X_2(s)$ is performed by separating the polynomial and trigonometric contributions. The term involving $\frac{2}{s^3}$ leads to polynomial components in the time domain, while the term $\frac{s(s-1)}{(s^2+1)(s-\lambda_1)(s-\lambda_2)}$ is decomposed as:

$$\frac{s(s-1)}{(s^2+1)(s-\lambda_1)(s-\lambda_2)} = \frac{As+B}{s^2+1} + \frac{C}{s-\lambda_1} + \frac{D}{s-\lambda_2},$$

with coefficients determined algebraically. For this problem, it is found that $A = 1/5$ and $B = -2/5$. **Consequently,**

$$X_2(s) = \frac{2}{s^3(s-\lambda_1)(s-\lambda_2)} - \left(\frac{1}{5}\cdot\frac{s}{s^2+1} - \frac{2}{5}\cdot\frac{1}{s^2+1} + \frac{C}{s-\lambda_1} + \frac{D}{s-\lambda_2}\right).$$

Under the given zero initial conditions, the exponential terms (corresponding to $C/(s-\lambda_1)$ and $D/(s-\lambda_2)$) vanish after simplification, leaving only the polynomial and trigonometric parts. Inverse Laplace transformation then yields:

$$x_2(t) = \frac{1}{3}t^2 - \frac{2}{3}t + \frac{1}{5}\cos t + \frac{2}{5}\sin t$$

For $x_1(t)$, use the relation $X_1(s) = \frac{s}{s^2+1} - (s-1)X_2(s)$. In the time domain, multiplication by $(s-1)$ corresponds to differentiation plus a negative shift: $\mathcal{L}^{-1}\{(s-1)X_2(s)\} = x_2'(t) - x_2(t)$. Hence,

$$x_1(t) = \cos t - (x_2'(t) - x_2(t)).$$

Computing $x_2'(t) = \frac{2}{3}t - \frac{2}{3} - \frac{1}{5}\sin t + \frac{2}{5}\cos t$ and substituting leads, after simplification**, to:**

$$x_1(t) = t^2 - 2t + 2 + \frac{1}{5}\cos t - \frac{2}{5}\sin t$$

Example 2: Exponential and Trigonometric Forcing with Nonzero Initial Conditions:

Consider the system:

$$\begin{cases} x_1' + 2x_1 + 3x_2 = e^{-t} \\ x_2' + 4x_1 + x_2 = \sin t \end{cases}$$

with initial conditions $x_1(0) = 1, x_2(0) = 0$.

Applying the Laplace transform to each equation gives:

$$\begin{cases} sX_1(s) - 1 + 2X_1(s) + 3X_2(s) = \dfrac{1}{s+1} \\ sX_2(s) - 0 + 4X_1(s) + X_2(s) = \dfrac{1}{s^2+1} \end{cases}$$

Rearranging yields:

$$\begin{cases} (s+2)X_1(s) + 3X_2(s) = 1 + \dfrac{1}{s+1} \\ 4X_1(s) + (s+1)X_2(s) = \dfrac{1}{s^2+1} \end{cases}$$

In matrix form,

$$\begin{pmatrix} s+2 & 3 \\ 4 & s+1 \end{pmatrix}\begin{pmatrix} X_1(s) \\ X_2(s) \end{pmatrix} = \begin{pmatrix} 1 + \dfrac{1}{s+1} \\ \dfrac{1}{s^2+1} \end{pmatrix}$$

The augmented matrix is:

$$\begin{pmatrix} s+2 & 3 & 1 + \dfrac{1}{s+1} \\ 4 & s+1 & \dfrac{1}{s^2+1} \end{pmatrix}$$

Applying Gauss-Jordan elimination transforms the augmented matrix into reduced row echelon form. The resulting expressions for the transformed variables are:

$$X_1(s) = \frac{12}{s+2} + \frac{3(11s-1)}{(s+5)(s+2)(s-2)}, X_2(s) = -4 - \frac{11s-1}{s^2+3s-10}$$

Factoring the denominator in the second term of $X_2(s)$ as $s^2 + 3s - 10 = (s+5)(s-2)$ enables partial fraction expansion. After performing the inverse Laplace transform, one obtains:

$$x_1(t) = 12e^{-2t} - 8e^{-5t} + \frac{23}{5}e^{2t} + \frac{9}{4}e^{-2t}$$

$$x_2(t) = -\delta(t) - (8e^{5t} + 3e^{-2t})$$

where $\delta(t)$ denotes the Dirac delta function arising from the constant term in $X_2(s)$.

Assessment of the Method:

The Laplace-transform-with-Gauss-Jordan approach offers several advantages. First, it converts the differential system into an algebraic one, thereby avoiding explicit integration. Second, it handles nonhomogeneous terms of polynomial, exponential, and trigonometric types uniformly through rational functions in the complex domain. Third, initial conditions are incorporated directly into the algebraic formulation, eliminating the need for separate determination of constants. Fourth, the method is systematic and algorithmic, making it suitable for implementation in computer algebra systems. Fifth, it provides insight into system dynamics through the resolvent operator $(sI - M)^{-1}$, whose poles (eigenvalues of M) determine stability and oscillatory behavior.

Nevertheless, limitations exist. The method requires a solid understanding of both Laplace transform theory and linear algebra, which may pose a barrier for novices. It is less effective for systems with time-varying coefficients (non-constant (t)) or for equations with discontinuous forcing terms, where alternative techniques such as the use of Green's functions may be more appropriate. Large systems lead to high-degree polynomial denominators, increasing the complexity of partial fraction decomposition and raising the risk of algebraic errors. Accuracy in performing inverse Laplace transforms is paramount, as mistakes in this step propagate directly to the final solution.

Conclusion:

This work has presented a unified algebraic framework for solving linear nonautonomous systems of differential equations by combining the Laplace transform with Gauss-Jordan elimination. The method transforms the original initial value problem $\mathrm{x}'(t) = \mathrm{Mx}(t) + \mathrm{h}(t), \mathrm{x}(0) = \mathrm{x}_0$, into the algebraic system $(s\mathrm{I} - \mathrm{M})\mathrm{X}(s) = \mathrm{x}_0 + \mathrm{H}(s)$. Solving this system via matrix row reduction and applying the inverse Laplace transform yields the time-domain solution without requiring explicit integration.

The theoretical significance of this approach lies in its demonstration of a structural equivalence between differential operators in the time domain and algebraic operators in the complex frequency domain. The resolvent operator $(s\mathrm{I} - \mathrm{M})^{-1}$ emerges as a central object that encodes both the intrinsic dynamics and the forced response. The method naturally decomposes the solution into homogeneous and particular parts, directly incorporates initial conditions, and provides a spectral characterization of the system's behavior through the eigenvalues of M. These features establish a deep connection between differential equations and linear algebra, offering both analytical power and computational convenience.

Practically, the method is advantageous for systems with constant coefficients and standard forcing functions, and it is particularly valuable in engineering, physics, and economics where initial value problems frequently arise. However, users must be mindful of its limitations, including mathematical complexity, reduced effectiveness for certain types of non-continuous or time-varying systems, and the potential for algebraic errors when dealing with large matrices or intricate partial fraction expansions. Future work may extend this approach to handle piecewise continuous inputs through the use of the Heaviside step function and the convolution theorem, thereby broadening its applicability.

In summary, the integration of Laplace transformation and Gauss-Jordan elimination constitutes a rigorous, systematic, and transparent methodology for solving linear nonautonomous systems. It not only provides a reliable computational procedure but also deepens the conceptual understanding of how algebraic properties of matrices govern the temporal evolution of dynamical systems.

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