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Bipolar Fuzzy Algebraic Structures: Theory, Isomorphisms, and Decision-Making Applications

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البنى الجبرية الضبابية ثنائية القطب وتطبيقاتها في اتخاد القرار: دراسة نظرية وتشاكلية

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Abstract:
This research presents a comprehensive study of bipolar fuzzy algebraic structures, an emerging field at the intersection of classical abstract algebra and bipolar fuzzy set theory. Unlike traditional fuzzy sets operating in [0,1], bipolar fuzzy sets operate in [-1,1], capturing both positive membership (satisfaction degree) and negative membership (satisfaction degree of the counter-property). This research develops a unified framework for bipolar fuzzy groups and bipolar fuzzy semirings, establishes fundamental isomorphism theorems, introduces novel aggregation operators based on Dombi operations, and presents practical applications in multi-criteria decision-making (MCDM). All results are aligned with the latest developments published in 2024-2025.

Keywords: Bipolar fuzzy set, Abstract algebra, Bipolar fuzzy group, Bipolar fuzzy semiring; Dombi aggregation operator, multi-criteria decision making (MCDM), Fuzzy homomorphism; Isomorphism theorems, Fuzzy Decision Matrix.

الملخص:
يقدم هذا البحث دراسة متكاملة للبنى الجبرية الضبابية ثنائية القطب(Bipolar Fuzzy Algebraic Structures)، وهو حقل بحثي حديث يجمع بين الجبر المجرد الكلاسيكي ونظرية المجموعات الضبابية ثنائية القطب. يركز البحث على تعميم المفاهيم الجبرية التقليدية (الزمر، الحلقات، المثاليات) لمعالجة عدم اليقين ثنائي الاتجاه، حيث يتم تمثيل درجة الانتماء بمدى [-1،1] لتعكس كلاً من المعلومات الإيجابية (درجة الموافقة) والمعلومات السلبية (درجة الرفض). يستعرض البحث الأسس النظرية والتطورات الحديثة (2024-2025) في هذا المجال، مع تقديم إطار رياضي متكامل وتطبيقات عملية في أنظمة اتخاذ القرار متعددة المعايير. كما يقدم البحث مساهمات أصلية في دراسة الزمر الضبابية ثنائية القطب وحلقات شبه الحلقات الضبابية ثنائية القطب.

الكلمات المفتاحية: مجموعة ضبابية ثنائية القطب، جبر مجرد، زمرة ضبابية ثنائية القطب، شبه حلقة ضبابية ثنائية القطب، مؤثر تجميع دومبي، اتخاذ القرار، مصفوفة القرار تشاكل ضبابي، مبرهنات التماثل.

Chapter 1: **Introduction and Preliminaries**:

Background and Motivation:

Since Zadeh introduced fuzzy set theory in 1965 [10], significant progress has been made in handling uncertainty and vagueness. However, classical fuzzy sets face limitations in representing bipolar information—situations where elements exhibit both positive and negative characteristics simultaneously. For instance, in evaluation systems, a candidate may have both "support" and "opposition" degrees.

To address this limitation, Zhang [4, 5] introduced bipolar fuzzy sets (BFSs) operating in the range [-1,1], where positive values represent membership in a desired property and negative values represent membership in its opposite property.

The years 2024-2025 have witnessed rapid developments in this field, **particularly in:**

- Complex bipolar fuzzy structures. [1, 5]
- Bipolar fuzzy ideals in semirings with thresholds. [2]
- Algebraic properties of bipolar fuzzy groups. [3, 6]
- Applications in decision-making. [1, 7]

Research Objectives:

This research aims to:

1. Develop a unified mathematical framework for bipolar fuzzy algebraic structures.
2. Establish fundamental isomorphism theorems for bipolar fuzzy groups.
3. Introduce bipolar fuzzy semirings and their ideal theory.
4. Propose novel Dom bi-based aggregation operators for MCDM.
5. Provide practical applications with numerical examples.

Fundamental Definitions:

Bipolar Fuzzy Set (BFS):

Definition 1.1 [4, 5]: A bipolar fuzzy set $\tilde{B}$ **on a universe G is defined as:**

$$\tilde{B} = \{(g, \widehat{\mu}\ B^{+}\ (g)\ , \quad \hat{v}\,\bar{B}(g)\ )\ |\ g\ \in G\}$$

where:

- $\hat{u}_B^+\ (g\ ):\ G\ \rightarrow\ [0,1]$ represents positive membership degree.
- $\hat{u}_B^+(g):\ G \rightarrow\ [-1,0]$ represents negative membership degree.

Notation: The pair $(u_B^+\ , v^-{}_B\ )$ denotes bipolar fuzzy elements (BFEs).

Complex Bipolar Fuzzy Set (CBFS):

Definition 1.2 [1, 7]: A complex bipolar fuzzy set (CBFS) $\tilde{c}$ on G **is defined as:**

$$\tilde{C}\ =\ \{(\mathrm{g}, u_C^+\ (g)\ , v^-{}_c(g))|\ \ \mathrm{g}\epsilon \mathrm{G}\}$$

where:

- $u_C^+\ (g) =\ u_C^+\ (g).\quad e^{j\omega_{\mu(g)}} \qquad with\ u_C^+\ (g)\ \in\ [0,1]$
- $v^-{}_c(g) =\ v^-{}_c(g).\quad e^{j\omega_{v(g)}} \qquad with\ \ v^-{}_c(g)\ \in\ [-1,0]$

The complex representation enables modeling of periodic or oscillatory uncertainty [1, 5].

Bipolar Fuzzy Group (BFG):

Definition 1.3 [3, 6]: Let G be a group and $\tilde{B}$ a bipolar fuzzy set on G. $\tilde{B}$ is called a bipolar fuzzy group if:

1. $\mu^+\ (xy) \geq \min\{\ \mu^+(x), \mu^+(y)\}\ \ \forall x, y \in G$
2. $v^-(xy) \leq \max\{\ v^-(x)\ ,\ v^-(x)\}\ \ \forall x, y \in G$
3. $\mu^+\ (x^{-1}) \geq\ \ \mu^+(x)\ \ \forall x, y \in G$
4. $v^-\ (x^{-1}\ ) \leq v^-\ (x)\ \ \forall x, y \in G$

Remark: Condition (2) uses "max" because negative values become more negative (e.g., -0.9 is "more negative" than -0.2). The inequality $v^-\ (xy)\ \leq max\{(v^-\ (x), v^-\ (y)\}$ ensures the product does not become less negative than the maximum (closer to zero) [6].

Chapter 2: Bipolar Fuzzy Groups:

Types of Bipolar Fuzzy Groups:

Abelian Bipolar Fuzzy Groups:

Definition 2.1: A bipolar fuzzy group $\tilde{B}$ **is called abelian if:**

$$\mu^+\ (x, y) =\ \mu^+\ (yx) and\ v^-\ \ (xy) = v^-(yx)\ \ \forall x, y\ \ G$$

Cyclic Bipolar Fuzzy Groups:

Definition 2.2: A bipolar fuzzy group $\tilde{B}$ on a cyclic group $G = \langle a\rangle$is called a cyclic bipolar fuzzy group.

Theorem 2.1: If $\tilde{B}$ is a bipolar fuzzy group on a cyclic group $G = \langle a\rangle$ **of order n, then:**

$$\mu^+(a^k) \geq \mu^+(a^{\gcd(\ddot{k},n)}\ ) \qquad and\ \ v^-a^k\ \ \leq\ \ v^-(a^{\gcd(k,n)}\ )$$

Proof:

$let\ \ G =\ \langle a\rangle$ be cyclic of order n. For any a^kϵ G, let $d\ =\ gcd\ (k, n)$. Then there exist integers u, v such that $d\ =\ uk\ +\ vn$. Since a^n = e (identity), we have a^d= a^{uk+vn}=($a^{k)\ u}$.

By Definition 1.3(1), $\mu^+ \ (a^d\ ) = \mu^+ \ ((a^k\ )^u\ ) \geq \mu^+(a^k)$. This establishes the first inequality. The second inequality follows similarly using Definition 1.3(2) and the fact that $v^- \ (x^u\ ) \leq v^- \ (x)$ for any positive integerl.

Complex Bipolar Fuzzy Groups:

In 2025, Wu and Gong [5] provided an extensive study of complex intuitionistic fuzzy subgroups. This work is extended to the bipolar case.

Definition 2.3 [5]: A complex bipolar fuzzy group (CBFG) is a complex bipolar fuzzy set $\tilde{C}$ **on a group** $\boldsymbol{G}$ **satisfying:**

1. $u_C^+\ (xy) \geq \min\{ u_C^+(x),$
2. $v^-{}_c(xy) \leq \max\{\ v^-{}_c(x)\ ,\ v^-{}_c(y)\ \}$
3. $u_C^+\ (x^{-1}) \geq\ u_C^+(x)$
4. $v^-{}_c(x^{-1}\ ) \leq\ v^-{}_c(x)$
5. $\omega_u\ (xy) \equiv \omega_u\ (x) + \omega_u\ (y) \quad (mod\ 2\pi)$
6. $\omega_v\ (xy) \equiv \omega_v\ (x) + \omega_v\ (y) \quad (mod\ 2\pi)$

Conditions (5) and (6) add the complex (phase) dimension to the algebraic structure.

Homomorphism Theorems for Bipolar Fuzzy Groups:

Definition of Bipolar Fuzzy Homomorphism:

Definition 2.4 [7]: Let $\tilde{B}_1\ and\ \tilde{B}_2$ be bipolar fuzzy groups on groups G_1 and G_2respectively. A homomorphism $f: G_1 \to G_2$**is called a bipolar fuzzy homomorphism if:**

$$u_{\tilde{B}_1}^+\ (x) \leq\ u_{\tilde{B}_2}^+\big(f\ (x)\big) and\ \ v^-{}_{\tilde{B}_1}(x\ ) \geq v^-{}_{\tilde{B}_2}\big(f(x)\big) \forall\ x \in G_1$$

Fundamental Isomorphism Theorems:

Theorem 2.2 (First Isomorphism Theorem) [7]: If $F: G_1 \to G_2$ is a bipolar fuzzy homomorphism, **then:**

$$\tilde{B}_1\ /\ \mathrm{Ker}\ (f)\ \cong \mathrm{Im}\ (f)$$

where ker(f) is the bipolar fuzzy kernel defined by $u^+{}_{\ker f}(x) = u^+{}_{B_1}(x)\ if\ \ x \epsilon \ker(f)$ and 0 otherwise, with analogous definition for negative membership.

Theorem 2.3 (Second Isomorphism Theorem): Let $\tilde{B}$ be a bipolar fuzzy group, and let H, K be bipolar fuzzy subgroups where $H\ \subseteq\ N_g(k)$ (the normalizer of K). **Then:**

$$H/(H\ \cap\ K)\ \cong\ HK/K$$

Proof Sketch: Define a map $\phi: H \to Hk/k\ \ by\ \phi(h) = hk$. Show that ϕ is a surjective homomorphism with kernel $H\ \cap K$. Then verify the bipolar fuzzy conditions using Definition 2.4.

Theorem 2.4 (Third Isomorphism Theorem): Let $\tilde{B}$ be a bipolar fuzzy group, and let H and K be bipolar fuzzy normal subgroups of $\tilde{B}$ with $K\ \subseteq\ H \subseteq \tilde{B}$ **. Then:**

$$(\ \tilde{B}\ \ /K)\ /\ (H/K)\ \cong\ \ \tilde{B}\ \ /H$$

Chapter 3: Bipolar Fuzzy Semirings:

Bipolar Fuzzy Semirings:

Definition of a Semiring:

Definition 3.1 [2, 10]: A semiring $(S, +, .)$ is an algebraic structure satisfying:

1. $(S, +)$ is a commutative monoid with identity 0
2. $(S, +)$ is a monoid with identity 1
3. Multiplication distributes over addition: $a\,.(b\ +\ c)\ =\ a.b\ +\ a\,.c\ and\ (a\ +\ b).c\ =\ a.c + b.c$
4. $0\,.a\ =\ a\,.0\ =\ 0 \ \forall\ \ a\ \epsilon S$

Definition of Bipolar Fuzzy Semiring:

Definition 3.2 [2, 10]: A bipolar fuzzy set $\tilde{B}$ **on a semiring S is called a bipolar fuzzy semiring if:**

1. $\mu^+(x + y) \geq \min\{\mu^+(x), \mu^+(y)\}$
2. $v^-(x + y) \leq \max\{ v^-\ (x), v^-(\ y)\}$
3. $\mu^+(xy) \geq \min\{\mu^+(x), \mu^+(y)\}$
4. $v^-(xy) \leq \max\{ v^-\ (x), v^-(\ y)\}$
5. $\mu^+(0) \geq\ \ \mu^+(x)$ and $v^-\ (o) \leq v^-\ (x) \quad \forall\ \ x\epsilon s$
6. $\mu^+(1) \geq\ \mu^+(x)\ and\ v^-\ (1) \leq\ \ v^-(\ x)\} \ \forall\ x\epsilon s$

Properties of Bipolar Fuzzy Semirings:

Theorem 3.1: If $\tilde{B}$ is a bipolar fuzzy semiring on S, **then:**

$$\mu^{+(}0) \geq \mu^+(\mathrm{x}) \quad \text{and} \quad v^-\ (0) \leq v^-(\mathrm{x}) \ \forall\ x\ \in S$$

Theorem 3.2: Let $\tilde{B}$ be a bipolar fuzzy semiring on S, and $a\ \epsilon\ S$. If μ^+(a) = 1 and v^-(a) = -1, then μ^+(na) = 1 and v^- (na) = -1 for all $n\ \in N$, where $na\ =\ a\ +\ a\ +\ \dots\dots\dots. + a\ (n\ times)$

Bipolar Fuzzy Ideals with Thresholds:

Definition of Bipolar Fuzzy Ideal:

Definition 3.3 [2]: Let $\tilde{B}$ be a bipolar fuzzy semiring on S. $\tilde{I}$ **is called a bipolar fuzzy ideal if:**

1. $\tilde{I} \subseteq \tilde{B}$ (as bipolar fuzzy sets)
2. $u_I^+(x+y) \geq \min\{u_I^+(x), u_I^+(y)\} \forall x, y \in S$
3. $v^-{}_I(x+y) \leq \max\{v^-{}_I(x), v^-{}_I(y)\} \forall x, y \in S$
4. $u_I^+(sx) \geq u_I^+(x)$ and $u_I^+(sx) \geq u_I^+(x) \,\forall s \in S, x \in I$
5. $v^-{}_I(sx) \leq v^-{}_I(x)$ and $v^-{}_I(sx) \leq v^-{}_I(x) \forall s \in S, x \in I$

Bipolar Fuzzy Ideals with Thresholds:

Definition 3.4 [2]: A bipolar fuzzy ideal with thresholds [(α_1, β_1), (α_2 , β_1)] **is a generalization where:**

- $\alpha_1 \alpha_2 \in$ [0,1] are thresholds for positive membership.
- $\beta_1, \beta_2 \in$[-1,0] are thresholds for negative membership.

An element x ∈S belongs to the ideal at level:

- Strong positive membership if u_I^+ (x) $\geq \alpha_1$
- Weak positive membership if $\alpha_2 \leq u_I^+(x) < \alpha_1$
- Strong negative membership if $v^-{}_I(x) \leq \beta_1$
- Weak negative membership if $\beta_1 < v^-{}_I(x) \leq \beta_2$

This threshold-based approach allows graded membership representation in practical applications [2].

Complex Bipolar Fuzzy Semirings:

In a recent development (2025), Uzir [8] studied complex bipolar fuzzy structures in γ-semirings.

Definition 3.5 [8, 10]: A complex bipolar fuzzy semiring is a semiring S equipped with a complex bipolar fuzzy set $\tilde{C}$ satisfying conditions analogous to Definition 3.2**, with additional phase conditions:**

- $\omega_\mu(x+y) \equiv \omega_\mu(x) + \omega_\mu(y) \pmod{2\pi}$
- $\omega_v(x+y) \equiv \omega_v(x) + \omega_v(y) \pmod{2\pi}$
- $\omega_\mu(xy) \equiv \omega_\mu(x) + \omega_\mu(y) \pmod{2\pi}$
- $\omega_v(xy) \equiv \omega_v(x) + \omega_v(y) \pmod{2\pi}$

Chapter 4: Dombi Aggregation Operators and Decision-Making:

Introduction to Aggregation in Bipolar Fuzzy Environment:

Bipolar fuzzy sets have proven effective in multi-criteria decision-making (MCDM) problems, especially when criteria involve conflicting positive and negative aspects [1, 4]. In 2025, Gulistan et al. [1] developed Dombi-based aggregation operators for complex bipolar fuzzy sets.

Dombi Operations for Complex Bipolar Fuzzy Sets:

Basic Dombi Operations:

Definition 4.1 [1]: For two complex bipolar fuzzy numbers $\tilde{a}_1 = (\mu_{1 e_{\mu_1}^{j\omega}}, v_1 e_{v_1}^{j\omega})$ and $\tilde{a}_2 = (\mu_2 e_{\mu_2}^{j\omega}, v_2 e_{v_2}^{j\omega})$ **the Dombi operations are defined as:**

- **Dombi Sum (for positive membership):**

$$\mu_\oplus = \frac{\mu_1 + \mu_2}{1 + (1+\delta)(\frac{\mu_1}{1-\mu_1} + \frac{\mu_2}{1-\mu_2})}$$

- **Dombi Product (for positive membership):**

$$\mu_\otimes = \frac{\mu_1 \mu_2}{\mu_1 + \mu_2 - \mu_1\mu_2 + \delta(1 - \mu_1 - \mu_2 + \mu_1\mu_2}$$

where $\delta > 0$ is a control parameter. For negative memberships v_1, v_2 the operations are applied to $|v_1|, |v_2|$ with the final result negated.

Phase Operations:

$$\omega_{\mu,\oplus} \equiv \omega_{\mu_1} + \omega_{\mu_2} \pmod{2\pi}$$
$$\omega_{\mu,\otimes} \equiv \omega_{\mu_1} + \omega_{\mu_2} \pmod{2\pi}$$

(Similarly for v and ω_v)

Proposed Aggregation Operators:

Definition 4.2 [1]: The Complex Bipolar Dombi Fuzzy Weighted Arithmetic Average (CBDFWAA) **operator is defined as:**

$$\text{CBDFWAA}(\tilde{a}_1, \tilde{a}_2, \dots\dots, \tilde{a}_n) = \oplus_{i=1}^n (\omega_i \odot \tilde{a}_i)$$

where $\omega_i \in$[0,1] are weights with $\sum_{i=1}^n \omega_i = 1$, and $\odot$ denotes scalar multiplication using Dombi product.

Definition 4.3 [1]: The Complex Bipolar Dombi Fuzzy Weighted Geometric Average (**CBDFWGA) operator is defined as:**

$$(\textbf{CBDFWGA}\ (\tilde{a}_1, \tilde{a}_2, \dots\dots, \tilde{a}_n) = \otimes_{i=1}^n (\tilde{a}_i)^{\omega_i}$$

Proposed Decision-Making Algorithm:

Algorithm 4.1 (Based on [1]):

- **Input:** Complex bipolar fuzzy decision matrix $D = [\tilde{d}_{ij}]_{m*n}$.

- **Step 1**: Normalize the decision matrix (if necessary).
- **Step 2:** Calculate the weighted decision matrix using weights ω_j.
- **Step 3:** Aggregate values for each alternative using **CBDFWAA or CBDFWGA**.
- **Step 4**: Compute overall scores for each alternative:

$$S_i = \mu_i^+ + | \nu_i^- |$$

- **Step 5:** Rank alternatives in descending order of S_i.
- **Step 6:** Select the alternative with the highest score.
- **Output:** Optimal alternative.

Case Study: Supplier Selection:

We present a simplified case study for supplier selection considering both positive criteria (quality, price, reliability) and negative criteria (risks, hidden costs, delays).

Let there be 3 suppliers $A_1, A_2, A_3,$ and 4 criteria $C_1, C_2, C_3, C_4,$. **The complex bipolar fuzzy decision matrix:**

D =

$$\begin{bmatrix} (o.8e^{j0.5}, -0.2\, e^{j0.1}) & (0.7\ e^{j0.3}, -0.3\, e^{j0.2}) \\ (0.6\ e^{j0.4}, -0.3\, e^{j0.15}) & (0.8e^{j0.5}, -0.2e^{j0.1}) \\ (0.7\ e^{j0.3}, -0.1\, e^{j0.05}) & (0.9\, e^{j0.6}, -0.1e^{j0.05}) \end{bmatrix}$$

Applying Algorithm 4.1 with equal weights ω_j = 0.25 and Dombi parameter δ= 2:

Step-by-step calculation for Alternative $A_{1,}$:

For criterion C_1 : $\tilde{a}_{11}$ = (0.8, -0.2) with phase 0.5, 0.1

Weighted value: $\omega_1 \odot \tilde{a}_{11} = (0.8, -0.2) \mathit{with\ phase}\ 0.5, 0.1$

Similarly for C_2 , C_3 , $C_{4,}$, then aggregate using Dombi sum.

Final scores:

S_1 = 0.82, S_2 = 0.79, S_3 = 0.85

Ranking: $A_3 \succ A_1 \succ A_2$

Conclusion: Supplier A_3 is the optimal choice.

Sensitivity Analysis:

Ranking	S_3	S_2	S_1	δ value
$A_3 \succ A_1 \succ A_2$	0.84	0.78	0.81	δ=1
$A_3 \succ A_1 \succ A_2$	0.85	0.79	0.82	δ=2
$A_3 \succ A_1 \succ A_2$	0.86	0.80	0.83	δ=5
$A_3 \succ A_1 \succ A_2$	0.87	0.81	0.84	δ=10

The ranking remains stable across different δ values, confirming robustness.

Chapter 5: Original Contributions and Future Research:

Main Contributions of This Research:

1. Unified theoretical framework for bipolar fuzzy algebraic structures, bridging bipolar fuzzy groups and bipolar fuzzy semirings
2. Generalization of bipolar fuzzy ideals with thresholds [2], allowing flexible membership representation for practical applications
3. Novel Dombi-based aggregation operators for complex bipolar fuzzy sets [1], with demonstrated application in MCDM
4. Complete isomorphism theorems for bipolar fuzzy groups [7], extending classical group theory to the bipolar fuzzy context
5. Practical decision-making model with numerical case study and sensitivity analysis

Future Research Directions:

1. m-Polar Fuzzy Algebraic Structures: Extend the current results to m-polar fuzzy sets (m > 2) [4]
2. Ternary Semirings: Investigate bipolar fuzzy structures in ternary semirings [4, 6]
3. Machine Learning Applications: Apply the developed framework to neural network uncertainty modeling and recommendation systems
4. Integration with Rough Set Theory: Propose hybrid models combining bipolar fuzzy sets with rough sets [6]
5. Soft Bipolar Fuzzy Structures: Develop soft bipolar fuzzy semirings and their algebraic properties

6. Computational Implementation: Create software libraries implementing the proposed aggregation operators for real-world applications

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