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Nonlinear Integer Programming Model for Solving Examination Timetable Problem

Houda Elmogassabi[1], Eman Miftah[2*]
[1] Industrial Engineering Department, Benghazi University, Benghazi, Libya
[2] Mechanical Engineering Department, Bright Star University, El–Brega, Libya

نموذج برمجة الأعداد الصحيحة غير الخطية لحل مشكلة جدول الامتحانات

هدى عبد الله علي المقصبي[1]، إيمان الشارف إبراهيم مفتاح[2*]
[1]قسم الهندسة الصناعية، جامعة بنغازي، بنغازي، ليبيا
[2] قسم الهندسة الميكانيكية، جامعة النجم الساطع، البريقة، ليبيا

*Corresponding author: eman.alsharif@bsu.edu.ly

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Abstract:
Educational timetabling is a crucial administrative task in most universities, often divided into course timetabling and examination timetabling problems. This work aims to develop a mathematical model that constructs an effective examination schedule for students in the Industrial and Manufacturing Systems Engineering Department at Benghazi University, Libya. The model includes extra constraints to avoid scheduling conflicts between courses of adjacent semesters, offering students increased flexibility during the enrollment process. A 0-1 nonlinear integer programming approach is utilized to assign sets of IE courses to a fixed number of time slots and rooms. This model was applied to four semesters: Fall 2016/2017, Fall 2017/2018, Spring 2017/2018, and Spring 2019/2020. Results indicate that the proposed model successfully reduces enrollment restrictions from three to two, offering students more options in course registration.

Keywords: Scheduling process, Examination timetabling, Nonlinear integer programming model.

الملخص
في معظم الجامعات، يُعتبر إعداد الجدول الزمني التعليمي عملاً مهمًا، وغالبًا ما يُقسّم إلى مشكلات جدول المقررات الدراسية وجدول الامتحانات. طوّرت هذه الدراسة نموذج رياضي لإنشاء جدول امتحانات فعّال لطلبة قسم الهندسة الصناعية ونظم التصنيف في جامعة بنغازي، ليبيا. حيث إن النموذج يحتوي على قيود إضافية لتفادي تعارض المواعيد بين الفصول المتتابعة، ويقدم المزيد من المرونة للطلاب خلال عملية التسجيل. في إطار تطوير النموذج تم التطرق لاستخدام طريقة *(0-1)*برمجة الأعداد الصحيحة غير الخطية لتعيين مجموعات من مقررات الهندسة الصناعية ونظم التنصنيع لعدد ثابت من الفترات الزمنية والقاعات. وتم تطبيق النموذج على أربعة فصول دراسية: خريف *2017/2016*، خريف *2018/2017*، ربيع *2018/2017*، ربيع *2020/2019*. أظهرت النتائج أن النموذج المقترح تمكن من تقليل قيود التسجيل بنجاح من ثلاثة قيود إلى قيدين، وأتاح للطلاب العديد من الخيارات في عملية التسجيل.

الكلمات المفتاحية: عملية الجدولة، جدول الامتحانات، نموذج برمجة الأعداد الصحيحة غير الخطية.

Introduction

Examination timetabling problem has been classified as an NP-hard optimization problem. It is hard to solve due to a large number of constraints having to be accommodated [1-3]. There are two major types of constraints to be satisfied when dealing with a university examination timetabling which are:

Hard and soft constraints. Hard constraints are conditions that must be conceited while soft constraints may not be conceited, but it is desirable to have a good and feasible timetable [3-7]. Several models exist which have been developed for the construction of a university course and examinations timetables. Birbas et al [1] developed a 0-1 nonlinear integer programming with objective of minimizing a linear cost function. The cost consisting of two terms. The first refers to the cost of assigning course m to the i th period of day i, while the second terms refer to the cost incurred from the assignment of those courses that require sessions of more than one consecutive hour, on a given day of the week.

The model was applied to different large-scale problems in different Greek universities and optimal solutions were obtained. Dimopoulou and Miliotis [2] described the design and implementation of a computer network-based system to aid the construction of a university course timetable. The proposed system used a centralized database, which contained both the flow of information among departments, staff, timetable constructors and end users and the actual construction of the timetable. The system used a nonlinear integer programming model to construct each department's timetable by assigning courses to time slots and rooms. The timetabling system has been tested with data provided by Athens university of economics and business. Ferland and Roy [3] presented a 0-1 mathematical programming approach to solve the course timetabling problem. The model decomposed the problem into two sub-problems; a class period scheduling problem and a classroom assignment problem. The approach was to first determine a class period schedule, then to repeatedly try to specify a feasible classroom assignment until such an assignment was obtained.

To deal with the problem effectually, Mandal and Kahar [7] used six graph heuristics to construct a feasible examination timetable then they used five trajectory algorithms to improve this feasible solution to find a more effective solution with high quality. Experiments have been tested on several instances of un-capacitated and capacitated benchmark datasets, which are Toronto and ITC2007 datasets, respectively. Abayomi-Alli et al [8] used particle swam optimization algorithm and local search technique to design an automatic examination timetable system for the Federal University of Agriculture, Abeokuta (FUNAAB), Nigeria. The examination timetabling results from this combination of the particle swarm optimization algorithm and local search technique were a near-optimal solution. The Examination timetable problem was proposed to be modeled as Constraint Satisfaction Problems (CSPs) and investigated by Constraint Logic Programming (CLP) approach in the research of Elsaka [9].

Elsaka used a real examination dataset from the Community College (CC), University of Sharjah (UoS). The objective of models by Kablan [5] and Kablan and Elsahali [6] are to develop an automated course timetabling system for the Industrial and Manufacturing Systems Engineering (IE) department, Benghazi University, Libya. The course timetabling problem was formulated as a 0-1 nonlinear Integer Programming model and solved by a heuristic algorithm developed to assign a set of courses to a fixed number of time periods taking into consideration the various primary and secondary constraints. Maitik et al [4] developed a heuristic algorithm to construct examination timetabling system based on enrollment of students in Industrial and Manufacturing Systems Engineering Department (IE), university of Benghazi, Libya during two semesters Fall 97/98 and Spring 97/98 and compared to the existing manual system.

Methodology

Problem description

At the beginning of each semester, a course timetable and an examination timetable are prepared together manually by an appointed committee consisting of several staff members. Students are then required to choose their courses based on both timetables such that these courses do not conflict with each other. In registration period, students are restricted by three restrictions (course timetable, examination timetable, and prerequisite courses). This study can reduce these restrictions on the students during courses enrollment to two restrictions (course timetable and prerequisite courses) and gives them more freedom in the choice of courses. Based on the characteristics of the examination timetabling procedures in the IE department, there are a set of constrains that are considered in the development of examination timetabling system. The following constrains have been taken into consideration during the scheduling process.

- Courses offered in the same semester are scheduled into time slots to ensure that no conflicts arise.
- Same semester courses should be spread over the examination period and should not be scheduled in adjacent time slots of time.
- Number of courses that are scheduled at any time slot must be less than number of rooms available.
- There is no conflict between courses in the seventh and eighth semesters.
- Pre-assignment courses, (certain courses are preassigned to specific time slots from other departments).

Notation

N	Number of courses
P	Number of time slots
r	The semester that has number of courses more than the half of number of time slots
CS_k	Set of courses in semester k
NS	Number of semesters
I	Course code (In Table 1)
R_l	Number of available rooms in time slot l
C_i	Set of courses in the next semester which considered course i is not its prerequisite course
H	Set of holidays time periods
PC	Set of preassigned courses

Decision variable

$$x_{il} = \begin{cases} 1 & \text{if course i is to be scheduled in time slot } l \\ 0 & \text{otherwise} \end{cases}$$

$$y_{rl} = \begin{cases} 1 & \text{if there is a course in the r}^{\text{th}} \text{ semester is scheduled in time slot } l \\ 0 & \text{otherwise} \end{cases}$$

Proposed model

A 0-1 Nonlinear Integer Programming will be used to formulate and solve the problem of examination timetable to produce a free conflict timetable. This algorithm assigns a set of courses to a fixed number of time slots. The required data include courses, and rooms available in each time slot. The 0-1 nonlinear integer programming problem is defined as follows.

The objective function is given as:

$$MAX \quad \sum_{l=1}^{P-1} y_{rl} \left(\sum_{i \in CS_r} x_{il} - 5 \sum_{i \in CS_r} x_{i,l+1} \right) \tag{1}$$

The constraints are as follows:

1. Courses scheduling constraints

$$\sum_{l=1}^{P} x_{il} = 1 \qquad i = 1,2, \dots \dots . N \tag{2}$$

2. Same semester courses constraints and spread constraints

$$\sum_{i \in CS_k} x_{il} + \sum_{i \in CS_k} x_{i,l+1} \leq 1 \qquad k=3, 5, \dots, NS \tag{3}$$

$$\sum_{i \in CS_r} x_{il} = y_{rl} \qquad l = 1, 2, \dots, P \tag{4}$$

3. Room assignment constraints

$$\sum_{i=1}^{N} x_{il} \leq R_l \qquad l = 1,,2, \dots, P \tag{5}$$

$$\sum_{i=1}^{N} x_{il} = 0 \qquad l \in H \tag{6}$$

4. Course pre-assignment constraints

$$x_{il} = 1 \qquad i \in PC, \forall l \tag{7}$$

5. Non-prerequisite courses constraints

$$(x_{il} + \sum_{j \in C_i} x_{j,l}) \leq 1 \qquad i = 1.2. \dots . N \qquad l = 1.2 \dots . P \qquad (8)$$

$$x_{il} = 0 \;\; or \; 1 \qquad \forall i \, , l$$
$$y_{rl} = 0 \;\; or \; 1 \qquad \forall l \, , r$$

Equation (1) represents the objective function that denotes the spread cost which is represented by the penalties assigned to time slots when the courses in the r[th] semester are scheduled in adjacent time slots. In Eq. (2), each course is scheduled on only one time slot. Eq. (3) shows that in the same semester courses, no more than one course is scheduled in a given time slot and any two courses should be spaced out by at least one time slot. Eq. (4) determines if there is any course in the r[th] semester is scheduled in time slot l. Eq. (5) explains that the number of courses scheduled in a given time slot never exceed the available number of rooms in this time slot. Eq. (6) restricts that no courses are scheduled in the holiday's periods. Eq. (7) ensures that certain courses (pre-assignment) are scheduled to specific time slot l. Then, Eq. (8) exhibits that each course should be scheduled without conflict with the courses in the next semester and provided that this course is not a prerequisite for any of them.

Implementation

This model is applied on data of four semesters: Fall 2016/2017, Fall 2017/2018, Spring 2017/2018, Spring 2019/2020. In Fall 2016/2017, the number of courses was 35 and the number of time slots was 15 time slots while in Spring 2019/2020, the number of courses was 36 and the number of time slots was 13. The number of courses and time slots in both semesters (Fall 2017/2018 and Spring 2017/2018) were the same numbers where they were 34 and 13, respectively. The change in the number of courses from one semester to another is due to the number of elective courses is offered in. Table 1 presents the courses taken in the same semester along with their prerequisites. Elective courses offered in semesters (Fall 2016/2017, Fall 2017/2018, Spring 2017/2018, Spring 2019/2020) are listed in Table 2. The pre-assignment courses are shown in Table 3 and they are scheduled outside the examination time slots in the first three semesters mentioned before thus they are not considered in the model but in the last semester, they are scheduled during the examination period therefore these courses are considered in the model. The number of rooms available was 4 rooms for each time slot.

Table 1: Courses in each semester.

3rd semester			4th semester		
Course Code i	**Course No.**	**Prerequisite**	**Course Code i**	**Course No.**	**Prerequisite**
1	IE203	ES102	7	IE204	ES201
2	IE227	ES102, ME102	8	IE212	ES201, IE203
3	ME209	ES122	9	IE224	ME209, ES217
4	ES201	ES102	10	IE228	IE227
5	ES217	ES102	11	IE250	ES102, ES261
6	ES261	ES102	12	ES206	ES102
-	-	-	13	ES218	ES217
5th semester			**6th semester**		
Course Code i	**Course No.**	**Prerequisite**	**Course Code i**	**Course No.**	**Prerequisite**
14	IE307	ES201, IE212	19	IE332	IE331
15	IE323	IE228		IE348	IE307
16	IE331	IE204, ES206	20	IE352	IE351
17	IE351	IE250	21	IE364	IE363
18	IE363	IE204	22	ME360	ES218
-	-	-	23	EE360	ES112
7th semester			**8th semester**		
Course Code i	**Course No.**	**Prerequisite**	**Course Code i**	**Course No.**	**Prerequisite**
24	IE403	IE332, IE351	30	IE402	IE307, IE403, IE409, IE464
25	IE409	IE332, IE351	31	IE406	IE352
26	IE422	IE351	32	IE414	ES261, IE409
27	IE428	IE323, EE360		IE449	IE448
28	IE464	IE351, IE363, IE364	33	Elective course 2	-
	IE448	IE348	34	Elective course 3	-
29	Elective course 1	-			

Table 2: Elective courses in the semesters

Semester	Elective courses
Fall 2016/2017	IE433-IE452-IE463-IE471
Fall 2017/2018	IE433-IE452-IE463-IE472
Spring 2017/2018	IE433-IE452-IE463-IE471
Spring 2019/2020	IE433-IE452-IE463-IE471-IE474

Table 3: Pre-assignment Courses

Pre-assignment Courses	Scheduled By
ES201, ES261, ES206	ES department

Result and Discussion

The optimized final examination timetable generated using LINGO software version 12 is summarized in Table 4. These results were obtained by applying a 0–1 nonlinear integer programming model on the Spring 2019/2020 semester data. As shown in Table 4, each course has been successfully assigned to a specific time slot. The objective function incorporates a spread cost, represented by penalties assigned to time slots where courses from the same semester are scheduled in consecutive time slots. This aims to reduce student fatigue and provide adequate time for study and rest between exams. This constraint was satisfied for all semesters, except for the fourth semester.

The absence of gaps between examinations in the fourth semester can be attributed to two main reasons. First, the fourth semester includes a relatively large number of courses (seven in total), which would require 14 to distribute exams evenly across days. However, only 12 time slots were available during the exam schedule, making it infeasible to avoid consecutive scheduling. Second, the scheduling of fourth-semester courses must be considered as potential conflicts with courses from the third and fifth semesters, further limiting scheduling flexibility for the fourth semester. For example, in the first week, IE224 and IE204 are scheduled on Tuesday and Wednesday, respectively. Similarly, in the second week, ES218 and IE228 follow the same consecutive pattern. These examples reflect the difficulty in avoiding consecutive exams for students in the fourth semester due to the constraints described above. Regarding room availability, the model also enforced the constraint that no more than four examinations could be scheduled in the same time slot, corresponding to the number of available rooms. For example, three time slots had the minimum number of exams (two exams), seven time slots had three exams, and two time slots reached full capacity with four exams scheduled simultaneously. Furthermore, Table 4 confirms that no courses were scheduled during designated holiday periods.

To avoid conflicts, each course was scheduled such that it did not overlap with courses in subsequent semesters—unless it was a prerequisite for one of them. For example, courses from the fourth semester (e.g., IE212, IE224, IE204) do not overlap with those from the fifth semester (e.g., IE331, IE363), except in cases where prerequisite relationships exist. For instance, IE250 was allowed to conflict with IE351 because IE250 is a prerequisite for that course. Overall, the results clearly demonstrate that the proposed model effectively satisfied all scheduling constraints.

Table 4: Industrial and Manufacturing Systems Engineering Department - Final Exams Time Table - Spring 2019/2020.

Date	Time	Semester					
		3	4	5	6	7	8
		Course	Course	Course	Course	Course	Course
SAT	9:00- 12:00			IE331		IE409	IE414
SUN	9:00 - 12:00	ES201	IE212		EE360		
MON	9:00 - 12:00			IE363		IE422	
TUE	9:00 - 12:00	ME209	IE224		IE352		IE474
WED	9:00 - 12:00		IE204			IE433	
THU	9:00 - 12:00	ES261		IE307			IE463
SAT	9:00 - 12:00	ES217	ES218		ME360		IE471
SUN	9:00 - 12:00		IE228	IE323		IE428	
MON	9:00 - 12:00	IE227			IE364		IE406
TUE	9:00 - 12:00		IE250	IE351		IE403	
WED	9:00 - 12:00		ES206		IE332		
THU	9:00 - 12:00	IE203				IE464	IE402

Conclusion and future work

In this study, optimization 0-1 nonlinear integer programming model was presented and applied on the IE department, Benghazi University, Benghazi, Libya in order to construct a conflict-free examination timetable. As a result, the main concluded points are:

1) This model gives more freedom to students in the choice of courses where they are restricted by only prerequisite courses and course timetable which released in the beginning of the semester by the existing manual system.
2) The development time for the proposed model is shorter than time spending on the manual approach to solving the examination timetable problem.
3) The results from running this model conclude and prove that the required constraints and the objective function are successfully verified.
4) This model prevents conflict between two successive semesters.
5) It's straightforward to modify and adapt the constraints, including variations in the number of courses, time slots, and prerequisites.
6) This model is capable of handling an increased number of constraints effectively.

For future work, the developed examination timetabling system should be used on the entire faculty of engineering because all engineering department use the same resources (rooms, timeslot). As a result, in this case, the constraints from pre-assignment courses will be eliminated.

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