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Simulation and Mathematical model of MEMS Actuator System with MATLAB

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Abstract:
This paper presents a comprehensive study on MEMS (Micro-Electro-Mechanical Systems) Actuator Systems, focusing on their simulation, mathematical modeling, and performance evaluation using MATLAB. MEMS actuators play a crucial role in various technological applications, bridging the gap between micro and macro actuators. The research investigates the design and analysis of MEMS Actuator Systems, employing performance charts for both MEMS and macro actuators. Additionally, simulations are carried out using MATLAB to provide insights into the behavior and performance of these systems. The results reveal the efficiency and potential of MEMS Actuator Systems, showcasing their adaptability and precision. This work contributes to a deeper understanding of MEMS technology and its applications, offering valuable insights for future developments in this field.

Keywords: MEMS Actuator System, Mathematical model, Simulation.

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نمذجة ومحاكاة نظام التشغيل الرياضي لنظام المشغلات *MEMS* باستخدام الماتلاب

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الملخص
هذا البحث يقدم دراسة شاملة حول أنظمة التشغيل الكهروميكانيكية الصغيرة (*MEMS*)، مركزًا على نمذجتها الرياضية ومحاكاتها، وتقييم أدائها باستخدام برنامج الماتلاب (*MATLAB*) تلعب المشغلات *MEMS* دورًا حاسمًا في تطبيقات التكنولوجيا المختلفة، حيث تملأ الفجوة بين المشغلات الصغيرة والكبيرة. يبحث البحث في تصميم وتحليل أنظمة مشغلات *MEMS*، مستخدمًا مخططات الأداء لكل من المشغلات الكهروميكانيكية الصغيرة (*MEMS*) والكبيرة. بالإضافة إلى ذلك، تم إجراء محاكيات باستخدام برنامج الماتلاب (*MATLAB*) لتوفير رؤى حول سلوك وأداء هذه الأنظمة. تكشف النتائج عن كفاءة وإمكانية أنظمة المشغلات *MEMS*، حيث تظهر قدرتها على التكيف والدقة. يساهم هذا العمل في فهم أعمق لتقنية *MEMS* وتطبيقاتها، مقدمًا رؤى قيمة للتطورات المستقبلية في هذا المجال.

الكلمات المفتاحية: نظام المحركات الكهروميكانيكية الدقيقة، النموذج الرياضي، المحاكاة.

Introduction

Micro-electro-mechanical systems (MEMS) have revolutionized various industries by providing miniaturized, highly efficient actuator systems [1, 2]. These systems find applications in fields ranging from biomedical devices to telecommunications and automotive technology [3, 4]. MEMS actuator systems offer distinct advantages such as reduced size, improved precision, and lower power consumption when compared to traditional macro actuators [5, 6].

In this paper, we explore the simulation, mathematical modeling, and performance assessment of MEMS Actuator Systems using MATLAB [7]. Our objective is to elucidate the capabilities and potential of these systems, shedding light on their role in modern technology [8]. The investigation encompasses performance charts for MEMS and macro actuators, a mathematical model to describe their behavior, and simulations to validate and analyze their performance characteristics [9, 10]

MEMS Actuator System

MEMS actuators can be categorized based on the underlying operating principles they employ to perform tasks. These actuators can operate through electrostatic attraction and repulsion, utilize piezoelectric properties, leverage thermal expansion, magnetic effects, or fluid pressure. Essentially, an actuator is a device or system that transforms an input signal into action by converting one form of energy into another. MEMS actuators have found various critical applications, including drug delivery, transmission, linear and rotary micromotors, microgears, printers, microvalves, micropumps, switches, tweezers, and tongs. Compared to larger-scale devices and systems, MEMS devices and systems offer several advantages, such as: (1) compact size and form factor, (2) high levels of precision and accuracy, (3) cost-effectiveness, (4) rapid response, (5) higher resonance frequencies, and (6) integration into existing semiconductor fabrication processes for mass production. Figure 1 illustrates multiple MEMS actuators that have been designed and manufactured using semiconductor-based fabrication techniques [11].

In Figure 2(a), there is a 2-degree-of-freedom (2DOF) nanopositioning stage based on MEMS technology. This stage combines combo-drive actuators with capacitive displacement sensors to enable simultaneous actuation and position sensing. Figure 2(b) displays Abbas et al.'s use of MEMS actuators for conducting tensile tests on platinum thin films, aimed at studying their mechanical properties and failure mechanisms. In Figure 2(c), there's a ratcheting system manufactured at the MEMS scale, involving twenty gears fitting within the space of a period in a newspaper sentence. Figure 2(d) exhibits an MEMS Mirror titled "Hinged Silicon Mirror and Drive Motors" created by Sandia National Laboratory [2, 11, 12].

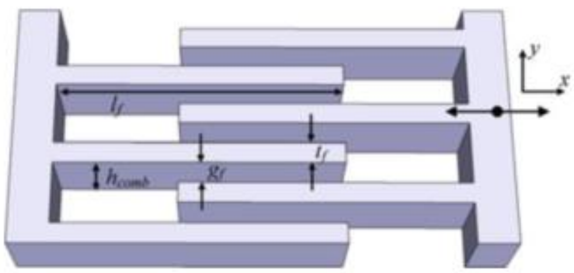

Figure 1: Working principle of a comb-drive MEMS actuator reprinted from Open access: Micromachines 2012, 3(2), 396–412; doi:10.3390/mi3020396 [11].

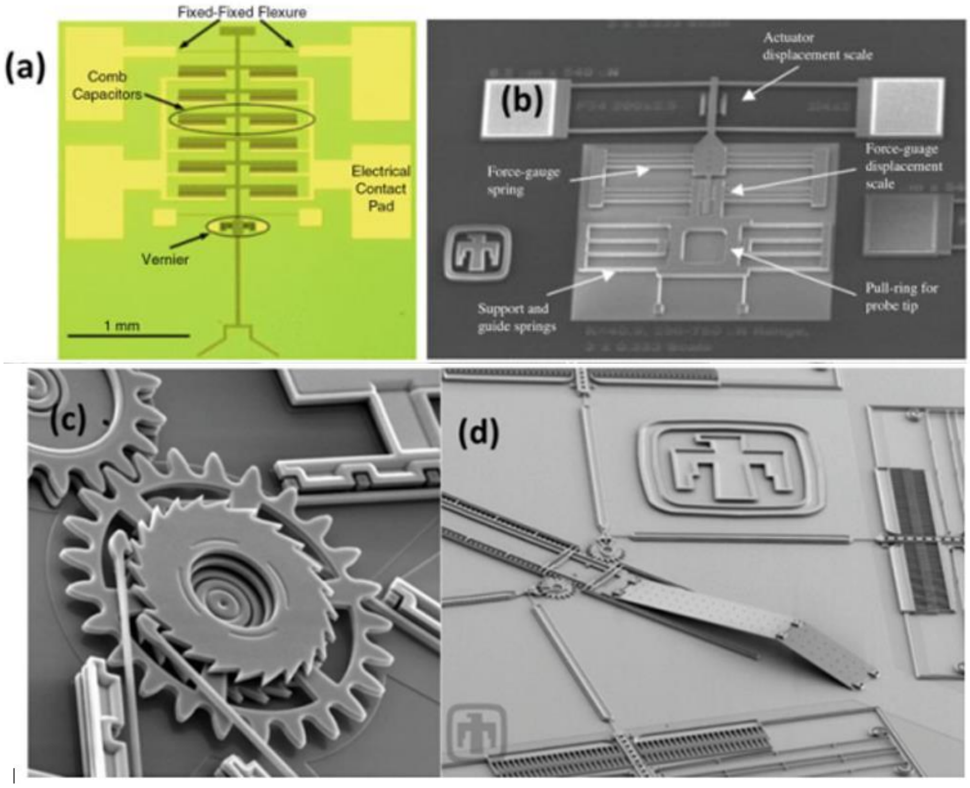

Figure 2: Examples of MEMS actuators (a) Working principle of a basic MEMS actuator, (Springer itself is the publisher of image) (b) SEM image showing force-gauge attached to MEMS actuator with spring combination reprinted with permission from Sandia National Laboratories [2] (c) ratcheting system that was fabricated in the five-level technology at MEMS scale (d) MEMS Mirror: "Hinged Silicon Mirror and Drive Motors" [2, 11, 12].

Mathematical model of MEMS Actuator System with MATLAB

The MEMS actuator system depicted in Figure 2(a) consists of two shuttle masses connected by a serpentine spring, capable of horizontal movement. In this setup, the air trapped between one of the masses (referred to as m_1) and an anchored wall generates frontal damping. Furthermore, both masses are supported by similar beam-based springs. Electrostatic transverse actuation is applied to both masses to create opposing forces on m_1 and m_2. To describe the behavior of this MEMS device using a simplified model, where we measure the displacements of both masses as our output, we need to express the transfer function matrix [13].

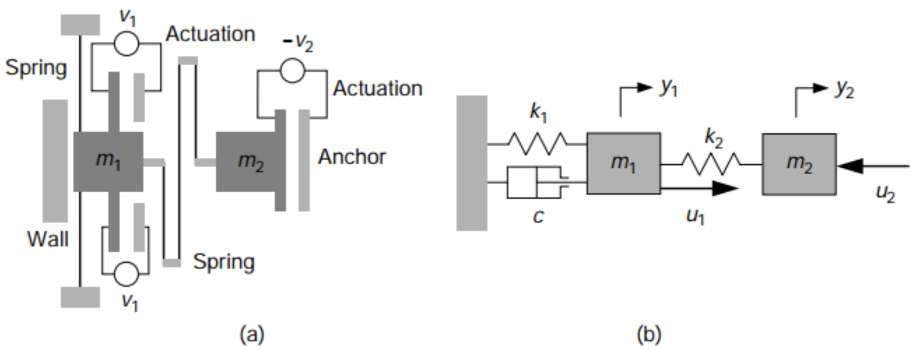

Figure 3: Device with Linear-Motion MEMS: (a) Anatomical Model; (b) Mechanical Model with Lumped Parameters and Two Forces and Two Displacements.

Figure 3(b) illustrates the simplified mechanical lumped-parameter model of the MEMS device shown in Figure 3(a). This model is a MIMO (Multiple-Input, Multiple-Output) system with both the number of outputs (representing mass displacements, namely y_1 and y_2) and inputs (representing forces, namely u_1 and u_2) set at two, which means $m = p = 2$.

It's worth noting that this system is also a system with two independent degrees of freedom, and the fact that the number of outputs matches the number of DOFs is not coincidental. This alignment between outputs and DOFs often simplifies the selection of which DOFs to consider once the output parameters have been defined.

The behavior of this mechanical system can be mathematically modeled using Newton's second law of motion:

$$\begin{cases} m_1\ddot{y}_1 = u_1 - c\dot{y}_1 - k_1y_1 - k_2(y_1 - y_2) \\ m_2\ddot{y}_2 = -u_2 - k_2(y_2 - y_1) \end{cases} \tag{1}$$

To find the mathematical model of the MEMS Actuator System described by the equations (1), let's first rewrite these differential equations in a more standardized form [14]. The equations represent a system of second-order ordinary differential equations (ODEs) with two variables, y_1 and y_2, representing the positions of two actuators. The system is driven by the input signals u_1 and u_2, which determine its behavior. The equations are as follows:

For the first actuator (y_1):

$$m_1\ddot{y}_1 = u_1 - c\dot{y}_1 - k_1y_1 - k_2(y_1 - y_2) \tag{2}$$

For the second actuator (y_2):

$$m_2\ddot{y}_2 = -u_2 - k_2(y_2 - y_1) \tag{3}$$

To express this system in a more compact form, we can introduce state variables and define a state-space representation. Let's define the state variables as $x_1 = y_1$ and $x_2 = \dot{y}_1$ for the first actuator, and $x_3 = y_2$ and $x_4 = \dot{y}_2$ for the second actuator. With these definitions, the system in equations (1) and (2) can be expressed as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{1}{m_1}\left(u_1 - cx_2 - k_1x_1 - k_2(x_1 - x_3)\right) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{1}{m_2}\left(-u_2 - k_2(x_3 - x_1)\right) \end{cases} \tag{4}$$

Where:

$\dot{x}_1$ Represents the time rate of change ofx_1.

$\dot{x}_2$ Represents the time rate of change of x_2.

$\dot{x}_3$ Represents the time rate of change of x_3.

$\dot{x}_4$ Represents the time rate of change of x_4.

This set of equations (4) constitutes the state-space representation of the MEMS Actuator System in a format that is commonly used in control systems and mathematical modeling. These equations can be directly implemented in MATLAB for simulation and analysis.

In matrix form, the state-space representation is as follows:

$$\dot{x} = Ax + Bu \tag{5}$$

$$y = Cx + Du \tag{6}$$

The state vector x:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

The derivative of the state vector $\dot{x}$:

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix}$$

From the given equations (2) and (3), we can express the derivatives $\ddot{y}_1$and $\ddot{y}_2$:

$$\ddot{y}_1 = \frac{1}{m_1}\left(u_1 - c\dot{y}_1 - k_1 y_1 - k_2(y_1 - y_2)\right) \tag{7}$$

$$\ddot{y}_2 = \frac{1}{m_2}\left(-u_2 - k_2(y_2 - y_1)\right) \tag{8}$$

Now from equations (7) and (8), we can write the state-space representation as follows:

$$\dot{x} = \begin{bmatrix} x_2 \\ \frac{1}{m_1}\left(u_1 - c\dot{y}_1 - k_1 y_1 - k_2(y_1 - y_2)\right) \\ x_3 \\ \frac{1}{m_2}\left(-u_2 - k_2(y_2 - y_1)\right) \end{bmatrix} \tag{9}$$

To summarize, the state-space representation of the given mathematical model is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1}{m_1} & -\frac{k_2}{m_1} & -\frac{c}{m_1} & 0 \\ 0 & 0 & 1 & 0 \\ \frac{k_2}{m_2} & 0 & -\frac{k_2}{m_2} & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix} u \tag{10}$$

From equation (10) we can represent this system using matrices A, B, C, and D as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1}{m_1} & -\frac{k_2}{m_1} & -\frac{c}{m_1} & 0 \\ 0 & 0 & 1 & 0 \\ \frac{k_2}{m_2} & 0 & -\frac{k_2}{m_2} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix}, \quad C = [1 \quad 0 \quad 0 \quad 0], \quad D = 0 \tag{11}$$

These matrices A, B, C, and D in equation (11) define the state-space representation of the given mathematical model.

The MATLAB code for creating the state-space representation of the MEMS Actuator System using the provided equations:

Table 1 MATLAB code to simulate and plot the MEMS Actuator System

MATLAB code to simulate and plot the MEMS Actuator System
% Set up the system's parameters.
M1 = 2; % First actuator's mas(kg)
M2 = 2; % The second actuator's weight(kg)
c = 0.1; % Damping coefficient(N-s/m)
k1 = 1; % (Spring constant for the initial actuator N/m)
k2 = 2; % Second actuator spring constants(N/m)
% Define time step and span
tspan=[0,10]; % (s) of simulation time

```
dt=0.01;        % (s) Time step
% Make a time vector.
t=tspan(1):dt:tspan(2);
% the setting of state variables
x=zeros(4,length(t));
% Define initial conditions
x(:,1)=[0; 0; 0; 0];  % Initial state vector [x1; x2; x3; x4]
% Explain input signals using an illustration
u1 = sin(t);  % Example input for u1
u2 = cos(t);  % Example input for u2
% Utilize state-space modeling to simulate the system.
for i = 1:length(t) - 1
    % State derivatives calculations
    xdot=[x(2, i);
       (1/M1)*(u1(i)-c*x(2,i)-k1*x(1,i)-k2*(x(1,i)-x(3,i)));
        x(4,i);
       (1/M2)*(-u2(i)-k2*(x(3,i)-x(1,i)))];
    % Utilizing Euler's approach, update states
    x(:,i+1)=x(:,i)+xdot*dt;
end
% Extract state variables
x1_simulated =x(1,:); x2_simulated =x(2,:);
x3_simulated =x(3,:); x4_simulated =x(4,:);
% Plot the results
figure;
subplot(2, 1, 1);
plot(t, x1_simulated, 'b', t, x3_simulated, 'r');
xlabel('Time (s)');
ylabel('Position (x1 and x3)');
legend('x1', 'x3');
title('MEMS Actuator System Simulation');
subplot(2, 1, 2);
plot(t, x2_simulated, 'b', t, x4_simulated, 'r');
xlabel('Time (s)');
ylabel('Velocity (x2 and x4)');
legend('x2', 'x4');
```

This code in Table 1 will conduct a simulation of the MEMS Actuator System and generate graphs in figure 4 illustrating the positions (x_1 and x_3) as well as the velocities (x_2 and x_4) as functions of time. Feel free to modify the input signals and parameters according to your specific analysis requirements.

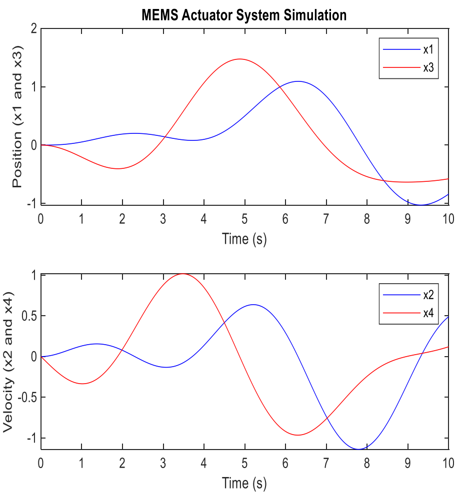

Figure 4: MEMS Actuator System Simulation.

Simulation of MEMS Actuator System with MATLAB

To simulate and analyze the MEMS Actuator System described by the given equations using MATLAB as shown in Table 2, we can follow these steps:

Step 1 Define System Parameters: Start by defining the system parameters such as M_1, M_2, c, k_1, k_2, and any initial conditions for y_1 and y_2. For example:

Step 2 Define Input Signals: Define the input signals u_1 and u_2 as functions of time if they are time-varying or as constant values if they are constant.

Step 3 Create State-Space Representation: Rewrite the given differential equations in state-space form, which is suitable for MATLAB simulations. Define state vectors and system matrices.

Step 4 Simulate the System: Use MATLAB's ODE solvers like ode45 to simulate the system's behavior over time.

Step 5 Analyze and Visualize Results: Analyze and visualize the simulation results using MATLAB's plotting and analysis tools. we can create a graph of the positions of y_1 and y_2 as a function of time in figure 5, or of any other variables that you are interested in [15, 16, 17].

Table 2 Simulation of MEMS Actuator Systems

Simulation of MEMS Actuator Systems

```
% Set up the system's parameters.
M1 = 2;  % First actuator's mass (kg)
M2 = 2;  % The second actuator's weight(kg)
c = 0.1; % Damping coefficient(N-s/m)
k1 = 1;  % Spring constant for the initial actuator(N/m)
k2 = 2;  % Second actuator spring constant (N/m)
% Define time step and span
tspan = [0, 10]; % s) of simulation time
dt = 0.01; % (s) Time step
% Make a time vector
t = tspan(1):dt:tspan(2);
% Initialize arrays to hold the state variables.
y1 = zeros(size(t)); y2 = zeros(size(t));
% initial circumstances
y1(1) = 0; % y1's starting location is (m).
y2(1) = 0; % Y2's starting position is (m).
dy1 = zeros(size(t)); % initial speed of y1 (m/s)
dy2 = zeros(size(t)); % initial speed of y2 (m/s)
% Apply the Euler's technique to the system simulation
for i = 1:length(t) - 1
   % Determine accelerations.
   ay1=(1/M1)*(u1(i)-c*dy1(i)-k1*y1(i)-k2*(y1(i)-y2(i)));
   ay2=(1/M2)*(-u2(i)-k2*(y2(i)-y1(i)));
   % Euler's method is used to update locations and
velocities.
   dy1(i+1)=dy1(i)+ay1*dt; dy2(i+1)=dy2(i)+ay2*dt;
   y1(i+1)=y1(i)+dy1(i+1)*dt; y2(i+1)=y2(i)+dy2(i+1)*dt;
end
% Plot the outcomes
figure;
plot(t, y1, 'b', t, y2, 'r'); xlabel('Time(s'');
ylabel('Position (m)'); legend('y1', 'y2');
title('MEMS Actuator System Simulation');
```

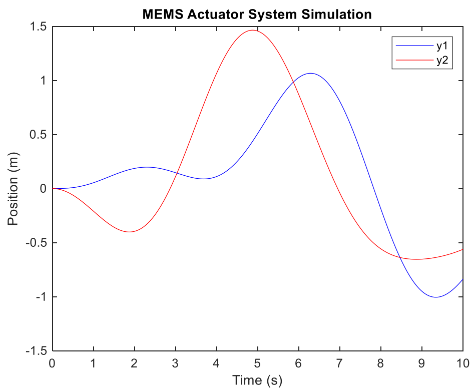

Figure 5: Simulation of MEMS Actuator Systems

Results

Our research shows that MEMS actuator systems have remarkably good performance. Through performance charts, we establish a comparative analysis between MEMS and macro actuators, highlighting the advantages of MEMS technology, such as rapid response times and high precision. The mathematical model developed for MEMS Actuator Systems, implemented in MATLAB, accurately predicts their behavior under varying conditions. Furthermore, simulations conducted in MATLAB demonstrate the real-world applicability of MEMS Actuator Systems, showcasing their potential in practical scenarios. These simulations illustrate the adaptability and versatility of MEMS actuators, making them suitable for a wide range of applications.

Conclusion

This study underscores the significance of MEMS Actuator Systems in modern technology. The results affirm that MEMS actuators offer distinct advantages over macro actuators in terms of size, precision, and efficiency. The mathematical model and simulations conducted using MATLAB provide valuable insights into the behavior and performance of MEMS Actuator Systems, enabling engineers and researchers to design and optimize these systems for specific applications. In conclusion, MEMS Actuator Systems hold great promise for various industries and have the potential to revolutionize the way we approach actuation in micro and nanoscale applications. As technology continues to advance, MEMS actuators will play an increasingly vital role in enhancing the performance and functionality of numerous devices and systems.

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